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Expected Time To Recruitment In A Two Grade Manpower System

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Abstract

In this paper a two graded organization is considered in which depletion of manpower occurs due to its policy decisions. Three mathematical models are constructed by assuming the loss of man-hours and the inter-decision times form an order statistics. Mean and variance of time to recruitment are obtained using an univariate recruitment policy based on shock model approach and the analytical results are numerically illustrated by assuming different distributions for the thresholds. The influence of the nodal parameters on the system characteristics is studied and relevant conclusions are presented.

Key words : Man power planning, Univariate recruitment policy, Mean and variance of the time for recruitment, Order statistics, Shock model.

I. Introduction

Exits of personnel which is in other words known as wastage, is an important aspect in the study of manpower planning. Many models have been discussed using different kinds of wastages and also different types of distributions for the loss of manhours, the threshold and the inter-decision times. Such models could be seen in [1] and [2]. Expected time to recruitment in a two graded system is obtained under different conditions for several models in [3],[4],[5],[6],[7],[8] and [9] according as the inter-decision times are independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables. Recently in [10] the author has obtained system characteristic for a single grade man-power system when the inter-decision times form an order statistics. The present paper extend the results of [10] for a two grade manpower system when the loss of man-hours and the inter decision times form an order statistics. The mean and variance of the time to recruitment of the system characteristic are obtained by taking the distribution of loss of man-hours as first order (minimum) and kth order (maximum) statistics respectively. This paper is organized as follows: In sections 2, 3 and 4 models I, II and III are described and analytical expressions for mean and variance of the time to recruitment are derived . Model I, II and III differ from each other in the following sense: While in model-I transfer of personnel between the two grades is permitted, in model-II this transfer is not permitted. In model-III the thresholds for the number of exits in the two grades are combined in order to provide a better allowable loss of manpower in the organization

compared to models I and II. In section 5, the analytical results are numerically illustrated and relevant conclusions are given.

II. Model description and analysis for Model-I

Consider an organization having two grades in which decisions are taken at random epochs in $[0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of man-hour to the organization, if a person quits and it is linear and cumulative. Let X_i be the loss of man-hours due to the ith decision epoch, i=1,2,3...k. Let $X_{i}, i=1,2,3...k$ are independent and identically distributed exponential random variables with density function g(.) and mean 1/c,(c>0). Let $X_{(1)}, X_{(2)}, \dots X_{(k)}$ be the order statistics selected from the sample $X_1, X_2, \dots X_k$ respective density with functions $g_{x(1)}(.), g_{x(2)}(.), \dots, g_{x(k)}(.)$. Let $U_i, i = 1, 2, 3 \dots k$ independent and identically distributed are exponential random variables with density function f(.). Let $U_{(1)}, U_{(2)}, ..., U_{(k)}$ be the order statistics selected from the sample $U_1, U_2, ..., U_k$ with respective density functions $f_{u(1)}(.), f_{u(2)}(.), \dots, f_{u(k)}(.)$. Let T be a continuous random variable denoting the time for recruitment in the organization with probability density function (distribution function) l(.)(L(.)).Let

 $l^*(.), f^*(.), f_{u(1)}^*(.)$ and $f_{u(k)}^*(.)$ be the Laplace transform of $l(.), f(.), f_{u(1)}(.)$ and $f_{u(k)}(.)$ respectively. Let Y_A and Y_B be independent random variables denoting the threshold levels for the loss of man-hours in grades A and B with parameters α_A and α_B respectively ($\alpha_A, \alpha_B > 0$). In this model the threshold Y for the loss of man-hours in the organization is taken as max (Y_A, Y_B). The loss of manpower process and the inter-decision time process are statistically independent. The univariate recruitment policy employed in this paper is as follows: **Recruitment is done as and when the cumulative loss of man-hours in the organization exceeds Y.** Let $V_k(t)$ be the probability that there are exactly k-decision epochs in (0,t]. Since the number of decisions made in (0,t] form a renewal process we note that $V_k(t) = F_k(t) - F_{k+1}(t)$, where $F_0(t)=1$. Let E(T) and V(T) be the mean and variance of time for recruitment respectively.

III. Main results

The survival function of T is given by

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(\sum_{i=1}^{k} X_i \le Y)$$

= $\sum_{k=0}^{\infty} V_k(t) \int_{0}^{\infty} p(y > x) g_k(x) dx$ (1)

Case 1:

 Y_A and Y_B follow exponential distribution with parameters α_A and α_B respectively. In this case it is shown

that

$$p(Y > x) = \sum_{k=0}^{\infty} V_k(t) \int_0^{\infty} \left(e^{-\alpha_A x} + e^{-\alpha_B x} - e^{-(\alpha_A + \alpha_B)x} \right) g_k(x) dx$$
(2)

From (1) and (2) we get

$$P(T > t) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t) \right] \left[g_k^*(\alpha_A) + g_k^*(\alpha_B) - g_k^*(\alpha_A + \alpha_B) \right]$$
(3)

Since
$$L(t) = 1 - P(T > t)$$
 and $l(t) = \frac{d}{dt}(l(t))$ (4)

from (3) and (4) it is found that

$$l(t) = [1 - g^{*}(\alpha_{A})] \sum_{k=1}^{\infty} f_{k}(t) (g^{*}(\alpha_{A}))^{k-1} + [1 - g^{*}(\alpha_{B})] \sum_{k=1}^{\infty} f_{k}(t) (g^{*}(\alpha_{B}))^{k-1} - [1 - g^{*}(\alpha_{A} + \alpha_{B})] \sum_{k=1}^{\infty} f_{k}(t) (g^{*}(\alpha_{A} + \alpha_{B}))^{k-1}$$

$$(5)$$

Taking Laplace transform on both sides of (5) it is found that

$$l^{*}(s) = \frac{\left[1 - g^{*}(\alpha_{A})\right]f^{*}(s)}{1 - f^{*}(s)g^{*}(\alpha_{A})} + \frac{\left[1 - g^{*}(\alpha_{B})\right]f^{*}(s)}{1 - f^{*}(s)g^{*}(\alpha_{B})} - \frac{\left[1 - g^{*}(\alpha_{A} + \alpha_{B})\right]f^{*}(s)}{1 - f^{*}(s)g^{*}(\alpha_{A} + \alpha_{B})}$$
(6)

The probability density function of r^{th} order statistics is given by

$$f_{u(r)}(t) = r k c_r [F(t)]^{r-1} f(t) [1 - F(t)]^{k-r}, r = 1, 2, 3. k$$
(7)

If $f(t)=f_{u(1)}(t)$

then
$$f^*(s) = f^*_{u(1)}(s)$$
 (8)
From (7) it is found that

 $f_{\mu(1)}(t) = k f(t) (1 - f(t))^{k-1}$ (9)

Since by hypothesis
$$f(t) = \lambda e^{-\lambda t}$$
 (10)

Since by hypothesis $f(t) = \lambda e^{-\lambda t}$ (10) from (9) and (10) we get

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$$f_{u(1)}^{*}(s) = \frac{k\lambda}{k\lambda + s}$$
(11)

It is known that

$$E(T) = -\frac{d(l^*(s))}{ds}\Big|_{s=0}, E(T^2) = \frac{d^2(l^*(s))}{ds^2}\Big|_{s=0} and V(T) = E(T^2) - (E(T))^2$$
(12)

Therefore from (6), (11) and (12) we get

$$E(T) = \frac{1}{\lambda} \left[V_1 + V_2 - V_3 \right]$$
(13)

$$E(T^{2}) = \frac{2}{\lambda^{2}} \left[V_{1}^{2} + V_{2}^{2} - V_{3}^{2} \right]$$
(14)

Where
$$V_1 = \frac{1}{1 - g^*(\alpha_A)}, V_2 = \frac{1}{1 - g^*(\alpha_B)} and V_3 = \frac{1}{1 - g^*(\alpha_A + \alpha_B)}$$
 (15)

If $f(t)=f_{u(k)}(t)$

k

In this case $f^*(s) = f_{u(k)}^*(s)$ From (7) it is found that $f_{u(k)}(t) = (F(t))^{k-1} f(t)$ (16)

From(10), (16) and on simplification we get

$$f_{u(k)}^{*}(s) = \frac{k!\lambda^{k}}{(s+\lambda)(s+2\lambda)\dots(s+k\lambda)}$$
(17)

Therefore from (6),(17) and (12) we get

$$E(T) = \frac{\sum_{n=1}^{\infty} \frac{1}{n}}{\lambda} \left[V_1 + V_2 - V_3 \right]$$
(18)

$$E(T^{2}) = \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2}}{\lambda^{2}} \left[\left(V_{1}^{2} - V_{1}\right) + \left(V_{2}^{2} - V_{2}\right) - \left(V_{3}^{2} - V_{3}\right) \right] + \frac{\sum_{n=1}^{k} \frac{1}{n^{2}}}{\lambda^{2}} \left[V_{1} + V_{2} - V_{3}\right]$$
(19)

In (18) & (19) V1, V_2 and V_3 are given by (15).

The probability density function of n^{th} order statistics is given by

$$g_{x(n)}(x) = nkc_n [G(x)]^{n-1} g(x) [1 - G(x)]^{k-n}, n = 1, 2, 3. k$$
(20)

If $g(x) = g_{x(n)}(x)$

If $g(x)=g_{x(1)}(x)$

then in(13),(14),(18) and (19) $g^*(\tau) = g^*_{x(1)}(\tau)$ for $\tau = \alpha_A, \alpha_B$ and $\alpha_A + \alpha_B$ From (20) it is found that

$$g_{x(1)}(x) = k g(x) (1 - g(x))^{k-1}$$
(21)

Since by hypothesis
$$g(x) = ce^{-cx}$$
 (22)

from (21) and (22) we get

$$g_{x(1)}^{*}(\tau) = \frac{kc}{kc+\tau}, \tau = \alpha_{A}, \alpha_{B} and \alpha_{A} + \alpha_{B}$$

$$(23)$$

In (13),(14),(18) and (19)
$$g^*(\alpha_A), g^*(\alpha_B) \& g^*(\alpha_A + \alpha_B)$$
 are given by (23) when s=1.
and $V(T) = E(T^2) - (E(T))^2$
If $g(\mathbf{x}) = g_{\mathbf{x}(\mathbf{k})}(\mathbf{x})$
then $g^*(\tau) = g^*_{\mathbf{x}(\mathbf{k})}(\tau)$ for $\tau = \alpha_A, \alpha_B$ and $\alpha_A + \alpha_B$

From (20) it is found that

$$g_{x(k)}(x) = (G(x))^{k-1} g(x)$$
From(22),(24) and on simplification we get
$$g_{x(k)}^{*}(\tau) = \frac{k! c^{k}}{(c+\tau)(2c+\tau)(3c+\tau)\dots(kc+\tau)} \text{ for } \tau = \alpha_{A}, \alpha_{B} \text{ and } \alpha_{A} + \alpha_{B}$$
(25)

In (13),(14),(18) and (19) $g^*(\alpha_A), g^*(\alpha_B) \& g^*(\alpha_A + \alpha_B)$ are given by (25) when s=k and $V(T) = E(T^2) - (E(T))^2$

Case 2:

 Y_A and Y_B follow extended exponential distribution with scale parameters α_A and α_B respectively and

shape parameter 2. In this case it can be shown that If $f(t)=f_{u(1)}(t)$

$$E(T) = \frac{1}{\lambda} \left[2V_1 + 2V_2 - 4V_3 + 2V_4 + 2V_5 - V_6 - V_7 - V_8 \right]$$
(26)

$$E(T^{2}) = \frac{2}{\lambda^{2}} \Big[2V_{1}^{2} + 2V_{2}^{2} - 4V_{3}^{2} + 2V_{4}^{2} + 2V_{5}^{2} - V_{6}^{2} - V_{7}^{2} - V_{8}^{2} \Big]$$
(27)

where
$$V_4 = \frac{1}{1 - g^*(2\alpha_A + \alpha_B)}, V_5 = \frac{2}{1 - g^*(\alpha_A + 2\alpha_B)}, V_6 = \frac{1}{1 - g^*(2\alpha_A + 2\alpha_B)}, V_{7=} \frac{1}{1 - g^*(2\alpha_A)}$$
 (28)

when n=1,in (26)&(27) V₁, V₂, V₃, V₄, V₅, V₆, V₇ and V₈ are given by (15),(28) and (23).

when n=k,in (26)&(27) $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ and V_8 are given by (15),(28) and(25). If $f(t)=f_{u(k)}(t)$

Proceeding as in case(i) it can be found that k

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} [2V_1 + 2V_2 - 4V_3 + 2V_4 + 2V_5 - V_6 - V_7 - V_8]$$

$$E(T^2) = \frac{2}{\lambda^2} [2V_1^2 + 2V_2^2 - 4V_3^2 + 2V_4^2 + 2V_5^2 - V_6^2 - V_7^2 - V_8^2] \left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \frac{1}{\lambda^2}$$
(29)

$$\left[2V_{1}+2V_{2}-4V_{3}+2V_{4}+2V_{5}-V_{6}-V_{7}-V_{8}\left[\left(\sum_{n=1}^{k}\frac{1}{n}\right)^{2}-\left(\sum_{n=1}^{k}\frac{1}{n^{2}}\right)\right]$$
(30)

when n=1,in (26)&(27) V₁,V₂,V₃,V₄,V₅,V₆,V₇ and V₈ are given by (15),(28) and (23). **when n=k**in (26) (27) V₁,V₂,V₃,V₄,V₅,V₆,V₇ and V₈ are given by (15),(28) and (25).

Case 3:

 Y_A follows extended exponential distribution with scale parameters α_A and shape parameter 2 and Y_B follows exponential distribution with parameter α_B . If $f(t)=f_{u(1)}(t)$ Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{\lambda} \Big[2V_1 + V_2 + V_4 - 2V_3 - V_7 \Big]$$

$$E(T^2) = \frac{2}{\lambda^2} \Big[2V_1^2 + V_2^2 + V_4^2 - 2V_3^2 - V_7^2 \Big]$$
(31)
(32)

when n=1, in (31) & (32) V₁, V₂, V₃, V₄ and V₇ are given by (15), (28) and (23).

when n=k, in (31) & (32) V₁, V₂, V₃, V₄ and V₇ are given by (15), (28) and (25).

If $f(t)=f_{u(k)}(t)$

Proceeding as in case (i) it can be shown that

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} [2V_1 + V_2 - 2V_3 + V_4 - V_7]$$
(33)

$$E(T^2) = \frac{2}{\lambda^2} [2V_1^2 + V_2^2 - 2V_3^2 + V_4^2 - V_7^2] \left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \frac{1}{\lambda^2} [2V_1 + V_2 - 2V_3 + V_4 - V_7] \left[\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \sum_{n=1}^{k} \frac{1}{n^2} \right]$$
(34)

when n=1, in (33) & (34) V₁, V₂, V₃, V₄ and V₇ are given by (15), (28) and (23).

when n=k, in (33) & (34) V₁, V₂, V₃, V₄ and V₇ are given by (15),(28) and (25). Case 4:

The distributions of Y_A has SCBZ property with parameters $\alpha_A, \mu_1 \& \mu_2$, and the distribution of Y_B has

SCBZ property with parameters $\alpha_B, \mu_3 \& \mu_4$. In this case it can be shown that **If** $f(t)=f_{u(1)}(t)$

$$E(T) = \frac{1}{\lambda} \left[p_1 V_9 + p_2 V_{10} - p_1 p_2 V_{13} - p_1 q_2 V_{14} - p_2 q_1 V_{15} - q_1 q_2 V_{16} + q_1 V_{11} + q_2 V_{12} \right]$$
(35)

$$E(T^{2}) = \frac{2}{\lambda^{2}} \Big[p_{1}V_{9}^{2} + p_{2}V_{10}^{2} - p_{1}p_{2}V_{13}^{2} - p_{1}q_{2}V_{14}^{2} - p_{2}q_{1}V_{15}^{2} - q_{1}q_{2}V_{16}^{2} + q_{1}V_{11}^{2} + q_{2}V_{12}^{2} \Big]$$
(36)

where

$$V_{9} = \frac{1}{1 - g^{*}(\alpha_{A} + \mu_{1})}, V_{10} = \frac{2}{1 - g^{*}(\alpha_{B} + \mu_{3})}, V_{11} = \frac{1}{1 - g^{*}(\mu_{2})}, V_{12} = \frac{1}{1 - g^{*}(\mu_{4})}$$

$$V_{13=} \frac{1}{1 - g^{*}(\alpha_{A} + \alpha_{B} + \mu_{1} + \mu_{3})}, V_{14} = \frac{1}{1 - g^{*}(\alpha_{A} + \mu_{1} + \mu_{4})}, V_{15} = \frac{1}{1 - g^{*}(\alpha_{B} + \mu_{1} + \mu_{3})}$$
and $V_{16} = \frac{1}{1 - g^{*}(\mu_{2} + \mu_{4})}$
(37)

when $n=1,in(35)\&(36)V_9,V_{10},V_{11},V_{12},V_{13},V_{14},V_{15}$ and V_{16} are given by (37) and (23).

when n=k,in(35)&(36)V₉, V₁₀, V₁₁, V₁₂, V₁₃, V₁₄, V₁₅ and V₁₆ are given by (37) and (25).

If $f(t)=f_{u(k)}(t)$

Proceeding as in case (i) it can be shown that k

$$E(T) = \frac{\sum_{n=1}^{\infty} \frac{1}{n}}{\lambda} \left[p_1 V_9 + p_2 V_{10} - p_1 p_2 V_{13} - p_1 q_2 V_{14} - p_2 q_1 V_{15} - q_1 q_2 V_{16} + q_1 V_{11} + q_2 V_{12} \right]$$
(38)

and

$$E(T^{2}) = \frac{2}{\lambda^{2}} \Big[p_{1}V_{9}^{2} + p_{2}V_{10}^{2} - p_{1}p_{2}V_{13}^{2} - p_{1}q_{2}V_{14}^{2} - p_{2}q_{1}V_{15}^{2} \Big] \Big(\sum_{n=1}^{k} \frac{1}{n} \Big)^{2} + \frac{2}{\lambda^{2}} \Big(\sum_{n=1}^{k} \frac{1}{n} \Big)^{2} \\ \Big[q_{1}V_{11}^{2} + q_{2}V_{12}^{2} - q_{1}q_{2}V_{16}^{2} \Big] - \frac{1}{\lambda^{2}} \Big[q_{1}V_{11} + q_{2}V_{12} - q_{1}q_{2}V_{16} \Big] \Big[\Big(\sum_{n=1}^{k} \frac{1}{n} \Big)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}} \Big] \\ - \frac{1}{\lambda^{2}} \Big[p_{1}M_{9} + p_{2}M_{10} - p_{1}p_{2}M_{13} - p_{1}q_{2}M_{14} - p_{2}q_{1}M_{15} \Big] \Big[\Big(\sum_{n=1}^{k} \frac{1}{n} \Big)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}} \Big]$$
(39)

when $n=1,in(35)\&(36)V_9,V_{10},V_{11},V_{12},V_{13},V_{14},V_{15}$ and V_{16} are given by (37) and (23). when $n=k,in(35)\&(36)V_9,V_{10},V_{11},V_{12},V_{13},V_{14},V_{15}$ and V_{16} are given by (37) and (25).

IV. Model description and analysis for Model-II

For this model $Y = \min(Y_A, Y_B)$. All the other assumptions and notations are

as in model-I. Then the values of $E(T) \& E(T^2)$ when r = 1 and r = k are given by case 1:

Lase 1: If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{\lambda} \begin{bmatrix} V_3 \end{bmatrix}$$

$$E(T^2) = \frac{2}{\lambda} \begin{bmatrix} V_2 \end{bmatrix}$$
(40)

$$E(T^2) = \frac{2}{\lambda^2} \left[V_3^2 \right]$$
(41)

when $n=1,in (40) \& (41) V_3$ is given by (15) and (23).

when n=k,in (40) & (41) V₃ is given by (15) and (25). If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that k

$$E(T) = \frac{\sum_{n=1}^{\infty} \frac{1}{n}}{\lambda} [V_3]$$
(42)

$$E(T^{2}) = \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2}}{\lambda^{2}} \left[V_{3}^{2}\right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}}{\lambda^{2}} \left[V_{3}\right]$$
(43)

when n=1, in (42) & (43) V₃ is given by (15) and (23).

when n=k, in (42) & (43) V₃ is given by (15) and (25). and $V(T) = E(T^2) - (E(T))^2$ Case 2: If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} \Big[4V_3 + V_6 - 2V_4 - 2V_5 \Big]$$
(44)

$$E(T^{2}) = \frac{2}{k^{2}\lambda^{2}} \Big[4V_{3}^{2} + V_{6}^{2} - 2V_{4}^{2} - 2V_{5}^{2} \Big]$$
(45)

when n=1, in (44) & (45) V₃, V₄, V₅ and V₆ are given by (15), (28) and (23).

when n=k,in (44) & (45) V₃,V₄,V₅ and V₆ are given by (15),(28) and (25). If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} [4V_3 + V_6 - 2V_4 - 2V_5]$$

$$E(T^2) = \frac{2}{\lambda^2} [4V_3^2 + V_6^2 - 2V_4^2 - 2V_5^2] \left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \frac{1}{\lambda^2} [4V_3 + V_6 - 2V_4 - 2V_5] \left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \sum_{n=1}^{k} \frac{1}{n^2}\right)$$
(46)
$$(46)$$

when n=1, in (46) & (47) V₃, V₄, V₅ and V₆ are given by (15), (28) and (23).

when n=k, in (46) & (47) V₃, V₄, V₅ and V₆ are given by (15), (28) and (25).

Case 3:

If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} [2V_3 - V_4]$$
(48)

$$E(T^{2}) = \frac{2}{k^{2}\lambda^{2}} \left[2V_{3}^{2} - V_{4}^{2} \right]$$
(49)

when n=1, in (48) & (49) V₃ and V₄ are given by (15), (28) and (23).

when n=k,in (48) & (49) V₃ and V₄ are given by (15),(28) and (25). If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} [2V_3 - V_4]$$
(50)

$$E(T^{2}) = \frac{2}{\lambda^{2}} \left[\left(2V_{3}^{2} - V_{4}^{2} \right) \left(\sum_{n=1}^{k} \frac{1}{n} \right)^{2} \right] - \frac{1}{\lambda^{2}} \left[\left(2V_{3} - V_{4} \right) \left(\sum_{n=1}^{k} \frac{1}{n} \right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}} \right]$$
(51)

when n=1,in (50) & (51) V₃ and V₄ are given by (15),(28) and (23).

when n=k,in (50) & (51) V₃ and V₄ are given by (15),(28) and (25). and $V(T) = E(T^2) - (E(T))^2$

Case 4:

If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} \Big[p_1 p_2 V_{13} + p_1 q_2 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16} \Big]$$
(52)

$$E(T^{2}) = \frac{2}{k^{2}\lambda^{2}} \Big[p_{1}p_{2}M_{13}^{2} + p_{1}q_{2}M_{14}^{2} + p_{2}q_{1}M_{15}^{2} + q_{1}q_{2}M_{16}^{2} \Big]$$
(53)

when n=1,in (52) & (53) V_{13} , V_{14} , V_{15} and V_{16} are given by (37) and (23).

when n=k,in (52) & (53) V_{13} , V_{14} , V_{15} and V_{16} are given by (37) and (25). If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

k . ,

$$E(T) = \frac{\sum_{n=1}^{k-1} n}{\lambda} [p_1 p_2 V_{13} + p_1 q_2 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16}]$$
(54)

$$E(T^2) = \frac{2}{\lambda^2} [p_1 p_2 V_{13} + p_1 q_2 V_{14} + p_2 q_1 V_{15} + q_1 q_2 V_{16}] \left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \left[\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2 - \sum_{n=1}^{k} \frac{1}{n^2}\right]$$
(55)

when n=1, in (54) & (55) V_{13} , V_{14} , V_{15} and V_{16} are given by (37) and (23).

when n=k,in (54) & (55) V_{13} , V_{14} , V_{15} and V_{16} are given by (37) and (25).

V. Model description and analysis for Model-III

For this model $Y = Y_A + Y_B$. All the other assumptions and notations are as in model-I. Then the values of $E(T) \& E(T^2)$ when n = 1 and n = k are given by case 1:

If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{\lambda} \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) V_2 - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) V_1 \right]$$

$$E(T^2) = \frac{2}{\lambda^2} \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) V_2^2 - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) V_1^2 \right]$$
(56)
(57)

when n=1,in (56) & (57) V₁ and V₂ are given by (15) and (23).

when n=k,in (56) & (57) V₁ and V₂ are given by (15) and (25). If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that k

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) V_2 - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) V_1 \right]$$

$$E(T^2) = \frac{2 \left(\sum_{n=1}^{k} \frac{1}{n} \right)^2}{\lambda^2} \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) V_2^2 - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) V_1^2 \right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n} \right)^2 - \sum_{n=1}^{k} \frac{1}{n^2}}{\lambda^2}$$

$$\left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) V_2 - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) V_1 \right]$$
(58)
(58)

when n=1,in (58) & (59) V₁ and V₂ are given by (15) and (23).

when n=k,in (58) & (59) V₁ and V₂ are given by (15) and (25).

Case 2: If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} \left[\left(\left(\frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right) V_1 + \left(\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right) V_2 \right] + \frac{1}{k\lambda} \left[\left(\left(\frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) \right) V_7 + \left(\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{\alpha_A - 2\alpha_B} \right) \right) V_8 \right]$$
(60)
$$E(T^2) = \frac{2}{k^2 \lambda^2} \left[\left(\left(\frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right) V_1^2 + \left(\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right) V_2^2 \right] + \frac{2}{k^2 \lambda^2} \left[\left(\left(\frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) \right) V_7^2 + \left(\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{\alpha_A - 2\alpha_B} \right) \right) V_8^2 \right]$$
(61)

when n=1,in (60) & (61) V₁, V₂, V₇ and V₈ are given by (15), (28) and (23).

when n=k,in (60) & (61) V₁, V₂, V₇ and V₈ are given by (15), (28) and (25).

If $f(t)=f_{u(k)}(t)$

k

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} \left[\left[\left(\frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_1 + \left[\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_2 \right] + \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} \left[\left[\left(\frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_7 + \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{\alpha_A - 2\alpha_B} \right) \right] V_8 \right]$$
(62)

$$E(T^2) = \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_1^2 + \left[\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_2^2 \right] + \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\frac{2\alpha_B}{2\alpha_A - \alpha_B} \right) - \left(\frac{\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_7^2 + \left[\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{\alpha_A - 2\alpha_B} \right) \right] V_8^2 \right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\frac{4\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_1 + \left[\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_2 \right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\frac{2\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_7 + \left(\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{4\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_2 \right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\left(\frac{2\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_7 + \left(\left(\frac{4\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_2 \right] \right] - \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^2}{\lambda^2} \left[\left[\left(\left(\frac{2\alpha_B}{\alpha_A - 2\alpha_B} \right) - \left(\frac{4\alpha_B}{\alpha_A - \alpha_B} \right) \right] V_7 + \left(\left(\frac{\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{2\alpha_A - \alpha_B} \right) \right] V_8 \right] \right]$$
(63)

when n=1,in (62) & (63) V₁, V₂, V₇ and V₈ are given by (15), (28) and (23).

when n=k,in (62) & (63) V₁, V₂, V₇ and V₈ are given by (15), (28) and (25).

Case 3: If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} \left[\left(\frac{\alpha_B}{2\alpha_A - \alpha_B} \right) V_7 - \left(\frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1 + \left(\left(\frac{2\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{2\alpha_A - \alpha_B} \right) \right) V_2 \right]$$
(64)

$$E(T^{2}) = \frac{2}{k^{2}\lambda^{2}} \left[\left(\left(\frac{2\alpha_{A}}{\alpha_{A} - \alpha_{B}} \right) - \left(\frac{2\alpha_{A}}{2\alpha_{A} - \alpha_{B}} \right) \right) V_{2}^{2} + \left(\frac{\alpha_{B}}{2\alpha_{A} - \alpha_{B}} \right) V_{7}^{2} - \left(\frac{2\alpha_{B}}{\alpha_{A} - \alpha_{B}} \right) V_{1}^{2} \right]$$
(65)

when n=1, in (64) & (65) V₁, V₂, V₇ and V₈ are given by (15), (28) and (23).

when n=k,in (64) & (65) V₁,V₂,V₇ and V₈ are given by (15),(28) and (25).

If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that $\frac{k}{k}$

$$E(T) = \frac{\sum_{n=1}^{N} n}{\lambda} \left[\left(\left(\frac{2\alpha_A}{\alpha_A - \alpha_B} \right) - \left(\frac{2\alpha_A}{2\alpha_A - \alpha_B} \right) \right) V_2 + \left(\frac{\alpha_B}{2\alpha_A - \alpha_B} \right) M_7 - \left(\frac{2\alpha_B}{\alpha_A - \alpha_B} \right) V_1 \right]$$
(66)

and

$$E(T^{2}) = \frac{2}{\lambda^{2}} \left[\left(\left(\frac{2\alpha_{A}}{\alpha_{A} - \alpha_{B}} \right) - \left(\frac{2\alpha_{A}}{2\alpha_{A} - \alpha_{B}} \right) \right) V_{2}^{2} + \left(\frac{\alpha_{B}}{2\alpha_{A} - \alpha_{B}} \right) V_{7}^{2} - \left(\frac{2\alpha_{B}}{\alpha_{A} - \alpha_{B}} \right) V_{1}^{2} \right] \left(\sum_{n=1}^{k} \frac{1}{n} \right)^{2} - \frac{1}{\lambda^{2}} \left[\left(\left(\frac{2\alpha_{A}}{\alpha_{A} - \alpha_{B}} \right) - \left(\frac{2\alpha_{A}}{2\alpha_{A} - \alpha_{B}} \right) \right) V_{2} + \left(\frac{\alpha_{B}}{2\alpha_{A} - \alpha_{B}} \right) V_{7} - \left(\frac{2\alpha_{B}}{\alpha_{A} - \alpha_{B}} \right) V_{1} \right] \left(\sum_{n=1}^{k} \frac{1}{n} \right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}} \right) (67)$$

when n=1,in (66) & (67) V₁, V₂, V₇ and V₈ are given by (15),(28) and (23).

when n=k,in (66) & (67) V₁, V₂, V₇ and V₈ are given by (15),(28) and (25). Case 4:

If $f(t)=f_{u(1)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{1}{k\lambda} \left[\left(\frac{p_1 p_2(\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right) V_{10} + \left(\frac{p_1 q_2(\alpha_A + \mu_1)}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{12} \right] - \frac{1}{k\lambda} \left[\left(\frac{p_1 p_2(\alpha_B + \mu_3)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{p_1 q_2 \mu_4}{\alpha_A + \mu_1 - \mu_4} \right) V_9 + \left(\frac{p_2 q_1(\alpha_B + \mu_3)}{\mu_2 - \mu_3 - \alpha_B} + \frac{q_1 q_2 \mu_4}{\mu_2 - \mu_4} \right) V_{11} \right]$$
(68)

and

$$E(T^{2}) = \frac{2}{k^{2}\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{q_{1}p_{2}\mu_{2}}{\mu_{2} - \alpha_{B} - \mu_{3}} \right) V_{10}^{2} + \left(\frac{p_{1}q_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} + \mu_{1} - \mu_{4}} + \frac{q_{1}q_{2}\mu_{2}}{\mu_{2} - \mu_{4}} \right) V_{12}^{2} \right] - \frac{2}{k^{2}\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{B} + \mu_{3})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{p_{1}q_{2}\mu_{4}}{\alpha_{A} + \mu_{1} - \mu_{4}} \right) V_{9}^{2} + \left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\mu_{2} - \mu_{3} - \alpha_{B}} + \frac{q_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}} \right) V_{11}^{2} \right]$$
(69)

when n=1, in (68) & (69) V₉, V₁₀, V₁₁ and V₁₂ are given by (28) and (23).

when n=k,in (68) & (69) V₉, V₁₀, V₁₁ and V₁₂ are given by (28) and (25).

If $f(t)=f_{u(k)}(t)$

Proceeding as in case 1 it can be shown that

$$E(T) = \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} \left[\left(\frac{p_1 p_2(\alpha_A + \mu_1)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{q_1 p_2 \mu_2}{\mu_2 - \alpha_B - \mu_3} \right) V_{10} + \left(\frac{p_1 q_2(\alpha_A + \mu_1)}{\alpha_A + \mu_1 - \mu_4} + \frac{q_1 q_2 \mu_2}{\mu_2 - \mu_4} \right) V_{12} \right] - \frac{\sum_{n=1}^{k} \frac{1}{n}}{\lambda} \left[\left(\frac{p_1 p_2(\alpha_B + \mu_3)}{\alpha_A - \alpha_B + \mu_1 - \mu_3} + \frac{p_1 q_2 \mu_4}{\alpha_A + \mu_1 - \mu_4} \right) V_9 + \left(\frac{p_2 q_1(\alpha_B + \mu_3)}{\mu_2 - \mu_3 - \alpha_B} + \frac{q_1 q_2 \mu_4}{\mu_2 - \mu_4} \right) V_{11} \right]$$
(70)

and

ŀ

$$E(T^{2}) = \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2}}{\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{q_{1}p_{2}\mu_{2}}{\mu_{2} - \alpha_{B} - \mu_{3}}\right) V_{10}^{2} + \left(\frac{p_{1}q_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} + \mu_{1} - \mu_{4}} + \frac{q_{1}q_{2}\mu_{2}}{\mu_{2} - \mu_{4}}\right) V_{12}^{2} \right] - \frac{2\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2}}{\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{B} + \mu_{3})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{p_{1}q_{2}\mu_{4}}{\alpha_{A} + \mu_{1} - \mu_{4}}\right) V_{9}^{2} + \left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\mu_{2} - \mu_{3} - \alpha_{B}} + \frac{q_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}}\right) V_{11}^{2} \right] - \frac{\left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}\right)}{\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{q_{1}p_{2}\mu_{2}}{\mu_{2} - \alpha_{B} - \mu_{3}}\right) V_{10} + \left(\frac{p_{1}q_{2}(\alpha_{A} + \mu_{1})}{\alpha_{A} + \mu_{1} - \mu_{4}} + \frac{q_{1}q_{2}\mu_{2}}{\mu_{2} - \mu_{4}}\right) V_{12} \right] + \frac{\left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}\right)}{\lambda^{2}} \left[\left(\frac{p_{1}p_{2}(\alpha_{B} + \mu_{3})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{p_{1}q_{2}\mu_{4}}{\mu_{2} - \alpha_{B} - \mu_{3}}\right) V_{9} \right] + \frac{\left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}\right)}{\lambda^{2}} \left[\left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{p_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}}\right) V_{9} \right] + \frac{\left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}\right)}{\lambda^{2}} \left[\left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\alpha_{A} - \alpha_{B} + \mu_{1} - \mu_{3}} + \frac{p_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}}\right) V_{9} \right] + \frac{\left(\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}\right)}{\lambda^{2}} \left[\left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\mu_{2} - \mu_{3} - \alpha_{B}} + \frac{p_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}}\right) V_{9} \right] + \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}}\right)}{\lambda^{2}} \left[\left(\frac{p_{2}q_{1}(\alpha_{B} + \mu_{3})}{\mu_{2} - \mu_{3} - \alpha_{B}} + \frac{p_{1}q_{2}\mu_{4}}{\mu_{2} - \mu_{4}}\right) V_{9} \right] + \frac{\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} - \sum_{n=1}^{k} \frac{1}{n^{2}}}\right)}{\lambda^{2}} \left[\left(\sum_{n=1}^{k} \frac{1}{n}\right)^{2} + \sum_{n=1}^{k} \frac{1}{n^{2}}}\right] \left[\frac{1}{n^{2}} + \frac{1}{n^{2}}\right) \left[\frac{1}{n^{2}} + \frac{1}{n^{2}}\right] \left[\frac{1}{n^{2}} + \frac{1}{n^{2}}\right] \left[\frac{1}{n^{2}} + \frac{1}{n^{2}}\right] \left[\frac{1}{n^{2}} + \frac{1}{n^{2}}\right] \left[\frac{1}{n^{2}} +$$

when n=1,in (70) & (71) V₉, V₁₀, V₁₁ and V₁₂ are given by (28) and (23).

when **n=k**,in (70) & (71) V₉, V₁₀, V₁₁ and V₁₂ are given by (28) and (25). and $V(T) = E(T^2) - (E(T))^2$

VI. Numerical illustration

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically. In the following tables these performance measures are calculated by varying the parameter ' ρ ' at a time and keeping the other parameters fixed as $\alpha A=0.1$, $\alpha B=0.3$, $\lambda=0.5$, $\mu 1=0.4$, $\mu 2=0.8$, $\mu 3=0.6$, $\mu 4=0.7$.

r										
с				1	1.5	2	1	1	1	
k				3	3	3	4	5	6	
		r=1	n=1	22.3333	33.1667	44	22.1667	22.0667	22	
Case 1	E(T)		n=k	4.4089	6.3780	8.3474	2.9331	2.1549	1.6829	
Cas		r=k	n=1	122.8333	182.41	242	184.7222	251.9278	323.4	
			n=k	24.4290	35.0788	45.9108	24.4423	24.6020	24.7386	
		r=1	n=1	31.8810	47.4881	63.0952	31.7143	31.6143	31.5476	
Case 2	E(T)		n=k	6.1437	8.9813	11.8190	4.0777	2.9902	2.3315	
Cas		r=k	n=1	175.3452	261.1845	347.0238	264.2857	360.9298	463.75	
			n=k	33.7902	49.3971	65.0042	33.9811	34.1387	34.2731	
		r=1	n=1	31.3333	46.6667	62	31.1667	31.0667	31	
Case 3	E(T)		n=k	6.0442	8.8319	11.6198	4.0121	2.9423	2.2943	
Cas			n=1	172.3333	256.6667	341	259.7222	354.6778	455.7000	
		r=k	n=k	33.2429	48.5754	63.9090	33.4338	33.5915	33.7262	
	E(T)	r=1	n=1	6.4309	9.3130	12.1951	6.2642	6.1642	6.0976	
Case 4			n=k	1.5244	2.0444	2.5668	1.0312	0.7680	0.6066	
			n=1	35.3700	51.2216	67.0733	52.2020	70.3751	89.6343	
				r=k	n=k	8.3840	11.2440	14.1171	8.5932	8.7677

Table 1: Effect of 'c' and 'k' on E(T) for Model-I

с				1	1.5	2	1	1	1
	I	<u>c</u>	1	3	3	3	4	5	6
	ie 1	r=1	n=1	5.6667	8.1667	10.6667	5.5	5.4	5.3333
Case 1			n=k	1.4041	1.8505	2.3001	0.9569	0.7170	0.5693
Cas	E(T)	r=k	n=1	31.1667	44.9167	58.6667	45.8333	61.65	78.40
			n=k	7.7227	10.1776	12.6536	7.9739	8.1853	8.3682
		r=1	n=1	9.4524	13.8452	18.2381	9.2857	9.1857	9.1190
Case 2	E(T)		n=k	2.0694	2.8662	3.6641	1.3897	1.0288	0.8086
Cas		r=1	n=1	51.9881	76.1488	100.3095	77.3810	104.8702	134.05
			n=k	11.3820	15.7642	20.1528	11.5808	11.7454	11.8864
		r=1	n=1	6.6667	9.6667	12.6667	6.5	6.4	6.3333
Case 3			n=k	1.5819	2.1190	2.6610	1.0715	0.9996	0.6327
Cas	E(T)	r=k	n=1	36.6667	53.1667	69.6667	54.1667	73.0667	93.1
			n=k	8.6910	11.6546	14.6355	8.9291	9.1287	9.3012
		r=1	n=1	2.5770	3.5322	4.4874	2.4104	2.3104	2.2437
4			n=k	0.8733	1.0311	1.1957	0.6149	0.4728	0.3834
Case 4	E(T)		n=1	14.1737	19.4272	24.6807	20.0864	26.3767	32.9824
		r=k	n=k	4.8031	5.6710	6.5765	5.1244	5.3976	5.6365

Table 2: Effect of 'c' and 'k' on E(T) for Model-II

с				1	1.5	2	1	1	1
k				3	3	3	4	5	6
			n=1	27.3333	40.6667	54	27.1667	27.0667	27
Case 1		r=1	n=k	5.3174	7.7413	10.2194	3.5325	2.5924	2.0226
ö	E(T)	r=k	n=1	150.3333	223.6667	297	226.3889	309.0111	396.900
			n=k	29.2459	42.5773	56.2069	29.4377	29.5963	29.7320
		r=1	n=1	40.6667	60.6667	80.6667	40.5	40.4	40.3333
Case 2	E(T)		n=k	7.7411	11.3775	15.0137	5.1320	3.7598	2.9292
Ca		r=1	n=1	223.6667	333.6667	443.6667	337.50	461.2333	592.900
			n=k	42.5758	62.5761	82.5755	42.7666	42.9246	43.0591
Case 3		r=1	n=1	37.3333	55.6667	74	37.1667	37.0667	37
	E(T)		n=k	7.1350	10.4684	13.8016	4.7320	3.4678	2.7024
		r=k	n=1	205.3333	306.1667	407	309.7222	423.1778	543.900
			n=k	39.2426	57.5761	75.9089	39.4333	39.5912	39.7258
			n=1	8.3413	12.1786	16.0159	8.1746	8.0746	8.0079
Case 4		r=1	n=k	1.8681	2.5635	3.2603	1.2570	0.9322	0.7336
Cas	E(T)	(T)	n=1	45.8770	66.9821	88.0873	68.1217	92.1851	117.717
		r=k	n=k	10.2747	14.0995	17.9314	10.4754	10.6425	10.7847

Table 3: Effect of 'c' and 'k' on E(T) for Model-III

Findings

From the above tables it is found that

- 1. When the probability density function of inter decision time is same as the probability density function of first order statistics, as 'k' increases the mean time to recruitment decreases for the first and kth order statistics for the loss of manhours but it is increases when the probability density function of inter decision time is same as the kth order statistics.
- 2. When the probability density function of inter decision time is same as the probability density function of first order statistics or the kth order statistics, as 'c' increases the mean time to recruitment increases for the first and kth order statistics for the loss of manhours.

Conclusion

Since the time to recruitment is more elongated in model-III than the first two models, model-III is preferable from the organization point of view.

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